Public Economics (ECON 131)
Section #5: Labor Income Taxation (Continued)

September 29 and October 4, 2016

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1 Labor Income Taxation

Public economists are interested in problems where the choice variables for the consumer are consumption and leisure. When this is the case, the budget constraint differs from the standard case in micro since income is no longer exogenous, but chosen by the consumer through their labor. In this section, we will consider how the budget constraint of the consumer changes in response to labor income taxes, and how this may affect choices of consumption and leisure.

1.1 Key concepts

- Income and substitution effects
- Earned Income Tax Credit
  - Understand what it is
  - Understand how it impacts the budget constraint
  - Know how to draw EITC budget constraint
  - Income and substitution effects on different portions of EITC budget constraint (Phase-in, flat, phase-out)

1.2 Practice problems

1.2.1 Labor income taxation with quasi-linear utility function

You graduate from UC Berkeley and take a job at a consulting firm with a wage of \$20 per hour. Your job is extremely flexible: You can choose any number of hours from 0 to 4000 per year. Your preferences over consumption (c) and leisure hours (h) are given by $u(c,h) = 100c - 0.5(4000 - h)^2$.

(a) Suppose there is a progressive income tax of the following form:

- *Income up to \$10,000: no tax*
- *Income from \$10,000 to \$40,000: 20% tax rate*
- *Income from \$40,000 up: 30% tax rate*

Draw a graph in consumption/leisure space showing your opportunity set with and without the tax system. Solve analytically for the optimal labor supply with the tax system.

(b) What is your marginal tax rate at this level of labor supply? What is your average tax rate? Do these rates differ? Why or why not?

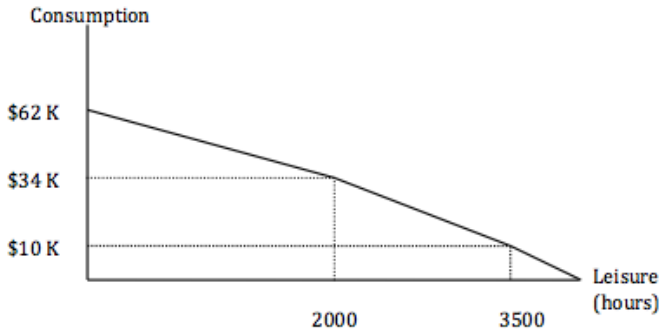
(c) Suppose that the highest income bracket is lowered so that the 30% tax rate begins to apply for incomes above \$30,000. How many hours will you choose to work now?

(d) Suppose that the government replaces the current tax system with a lump-sum tax: each person pays \$10,000 per year in taxes regardless of what they earn. Draw your new opportunity set. What is your new labor supply? What is the deadweight loss associated with this tax?

(e) With this "quasi-linear" utility function, do changes in the tax rate affect labor supply through a price effect, income effect, or both?

Solution:

(a) The (non-linear) budget constraint is shown below:



The optimization problem: $\max u(c,h) = 100c - 0.5(4000-h)^2$ subject to the budget constraints:

$$\begin{aligned} c &= 20(4000-h) && \text{for} \quad 3500 \leq h \leq 4000 \\ c &= 16(3500-h)+10,000 && \text{for} \quad 2000 \leq h < 3500 \\ c &= 14(2000-h)+34,000 && \text{for} \quad 0 \leq h < 2000 \end{aligned}$$

To solve this non-linear budget constraint optimization problem, optimize on each part of the budget constraint separately, pretending that this budget constraint applies for all levels of h.

Graphically, extend out each segment to zero and infinity and pretend that the budget constraint is just that line in each case.

To maximize on each segment, simply plug in for c using the budget constraint above and solve the calculus problem by taking a derivative and setting it equal to zero. Of the feasible solutions (the ones that lie within the relevant segment), choose the one that gives highest utility.

Solving the calculus problem implies that on each segment with a different marginal rate t_i, where t_i denotes the marginal tax rate in tax bracket $i = 1, 2, 3$, the optimal h is found by setting

$$\frac{MU_c}{MU_h} = \frac{1}{w_{net}}, \quad \text{where} \quad w_{net} = 20(1-t_i)$$

Segment one:

- Optimization: $\frac{MU_c}{MU_h} = \frac{1}{w_{net}} \Longrightarrow \frac{100}{4000-h} = \frac{1}{20}$
- Solving implies that the individual would choose to work 2000 hours at this marginal rate.
- However, the maximum number of hours the individual can work and still be on the first segment of the budget constraint is $\frac{\$1000}{\$20} = 500$ hours, so this solution is not feasible.

Segment two:

- Optimization: $\frac{MU_c}{MU_h} = \frac{1}{w_{net}} \Longrightarrow \frac{100}{4000-h} = \frac{1}{20(1-0.2)}$
- Solving implies that the individual would choose to work 1600 hours at this marginal rate.
- This gives taxable income of $1600 * 20 = \$32,000$.
- This is within the second tax bracket, so this solution is feasible.

Segment three:

- Optimization: $\frac{MU_c}{MU_h} = \frac{1}{w_{net}} \Longrightarrow \frac{100}{4000-h} = \frac{1}{20(1-0.3)}$
- Solving implies that the individual would choose to work 1400 hours at this marginal rate.
- Now we can see that this solution is not feasible either.

The individual will maximize utility by working 1600 hours.

(b) The marginal tax rate faced by the individual is just the tax rate of the second bracket, 20%.

The average tax rate can be computed using gross (before tax) and net (after tax) earnings.

- If the individual was not taxed, he would earn $\$20 * 1600 = \$32,000$.
- After tax he earns $500 * \$20 + (1600 - 500) * \$20 * (1 - 0.2) = \$27,600$.
- This gives the following: $\$32,000 * (1 - t_{average}) = \$27,600$.
- Which yields an average tax rate of 13.75%.

The marginal tax rate is larger than the average tax rate. This is because the individual is not taxed on the first $10,000 of earnings, so they are not paying 20% on total earnings.

(c) If the top tax bracket (30%) shifts down to $30,000, we perform the same maximization as in part (a) but with the new budget constraint.

Segment two:

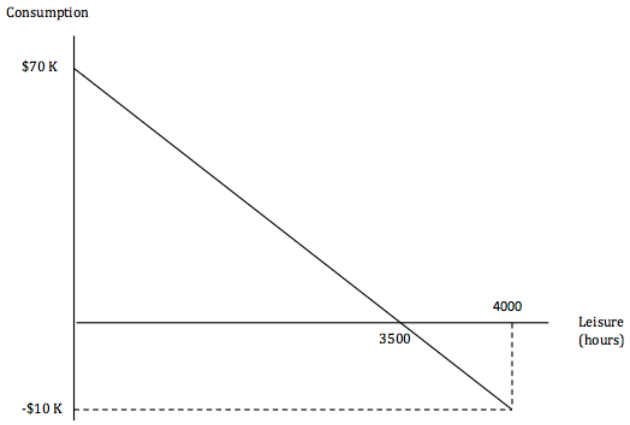
- Optimization: $\frac{MU_c}{MU_h} = \frac{1}{w_{net}} \Longrightarrow \frac{100}{4000-h} = \frac{1}{20(1-0.2)}$
- Solving implies that the individual would choose to work 1600 hours at this marginal rate.
- This gives taxable income of $1600 * 20 = \$32,000$.
- Now the optimal earnings are *too high* to remain in this tax bracket.

Segment three:

- Optimization: $\frac{MU_c}{MU_h} = \frac{1}{w_{net}} \Longrightarrow \frac{100}{4000-h} = \frac{1}{20(1-0.3)}$
- Solving implies that the individual would choose to work 1400 hours at this marginal rate.
- This gives taxable income of $1400 * 20 = \$28,000$.
- Now the optimal earnings are *too low* to remain in the top tax bracket.

Therefore, the individual will locate at the convex kink between tax bracket 2 and tax bracket 3, working $\$30,000/\$20 = 1500$ hours.

(d) New budget constraint with lump-sum tax of $10,000:



When there is a lump sum tax of T dollars, the individual chooses labor supply by solving $\max u(c, l) = 100c - 0.5(4000 - h)^2$ subject to the budget constraint $c = 20 * (4000 - h) - T$.

- Optimization: $\frac{MU_c}{MU_h} = \frac{1}{w_{net}} \Longrightarrow \frac{100}{4000-h} = \frac{1}{20}$
- Solving implies that the individual would choose to work 2000 hours.
- The labor supply in this case is the same number of hours the individual would choose to work for segment one, the no tax case.
- The lump-sum tax does not distort the labor supply decision, it is the same as if there was no tax at all.
- If the government were to return $T back to the individual after taxing him $T, he would be exactly as well of as he was without the tax, which means there is no DWL.

(e) With this "quasi-linear" (linear in consumption c) utility function, changes in tax rate affect labor supply only through substitution (or price) effects and not through income effects.

- In part (d): The lump sum tax shifts income down at every choice of leisure, but the individual does not change his behavior in response to this income effect and would choose to work the same amount of hours as in the case of no taxes. His behavior only changes when the net wage changes.
- In part (c): The net wage rate changes for the worker from part (a). Since work pays less now at the margin (the marginal tax rate faced by the worker increased from 20% to 30%), the worker unambiguously decreases his labor supply.

1.3 Additional problems for practice

1.3.1 Gruber, Ch.21, Q.5

The country of Akerlovia currently has a tax system that gives each citizen $5,000 in cash up front, exempts the first $10,000 in earned income from tax, and taxes all earned income over $10,000 at a 25% rate. It is considering replacing this system with an Earned Income Tax Credit system. The proposed new system would drop the $5,000 cash give-away and would instead subsidize the first $10,000 in earned income at a 50% rate. All income earned over $10,000 would still be taxed at the same 25% rate, and the EITC benefits would never be phased out. Describe the effects of this policy change on the labor supply of workers with various incomes.

Solution:

- This policy change has no effect on any worker with income over $10,000. Under either system, these workers get $5,000 from the government and they face a 25% marginal tax rate.
- It will encourage work among all other workers via both the income and substitution effects.
- At their original levels of work, these workers will be poorer under the new system as they will get less than $5,000 from the government, so the income effect will encourage them to work more (consume less leisure).
- Furthermore, an additional hour of work now yields a 50% larger increase in their take-home income than it did under the old system. Hence, the substitution effect also leads them to work more.

1.3.2 Gruber, Ch.21, Q.8

Suppose that the government introduces an Earned Income Tax Credit such that for the first $8,000 in earnings, the government pays 50¢ per dollar on wages earned. For the next $3,000 of earnings, the credit is held constant at $4,000, and after that point the credit is reduced at a rate of 20¢ per dollar earned. When the credit reaches zero, there is no additional EITC.

(a) Draw the budget constraint that reflects this earned income tax credit for a worker who can work up to 4,000 hours per year at an hourly wage of $10 per hour.

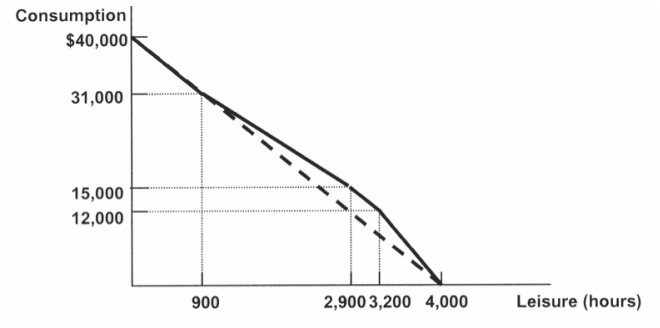
(b) Illustrate on your graph the portions of the budget constraint where the labor supply effects of the policy are positive, negative, or ambiguous, relative to the "no policy" status quo.

Solution:

(a) The 50¢ subsidy applies to the first $8,000 of earnings, or the first 800 hours of work. This corresponds to 3,200 hours of leisure and a consumption of $12,000.

The next $3,000 of earnings, or 300 hours of labor, is untaxed. Hence, at 2,900 hours of leisure, the worker gets a consumption of $15,000.

The $4,000 EITC benefit is phased out gradually, disappearing after $20,000 in additional earnings. Hence, at 900 hours of leisure, the worker gets to consume $31,000.



(b) Budget constraint with the different labor supply effects: